# AN INVESTIGATION INTO THE RATE OF MATERIAL REMOVAL IN ULTRASONIC MACHINING

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY



BY
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to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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# CERTIFICATE

This is to certify that the work "An investigation into the rate of material removal in ultrasonic machining" by Amitava Nandy has been carried out under my supervision and has not been submitted elsewhere for a degree.

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A. NANDY

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## SYNOPSIS

The present dissertation deals with the analysis of the rate of material removal in ultrasonic machining considering the direct impact of abrasive grains on the workpiece. The non-uniformity in the size of the abrasive grains has been taken into consideration by using a statistical distribution for the diameter of the abrasive particles, assumed to be spherical. A general expression for the rate of material removal due to the active grains taking part in the process has been obtained.

Experiments have been conducted on an Ultrasonic Drilling Machine to study the effect of frequency, static force and concentration of abrasive slurry on the cutting rate. Theoretically calculated values have been compared with the experimental results.

# CHAPTER I

## INTRODUCTION

Current techniques for machining of common materials are highly developed. In recent years, machine tools have been greatly improved with the result that it is possible to solve many of the varied and complex problems raised by rapid advances in technology.

However, materials like tungsten and titanium carbides, diamonds, hard steels, magnetic alloys and corundum, being very hard and strong, are difficult to machine. Materials like germanium, silicon, ferrites, ceramics, glass and quartz, are unable to withstand the forces needed for machining due to their brittleness. Ultrasonic machining is one of the methods used for machining such materials.

In ultrasonic machining, linear vibrations are produced in a tool by a transducer. The tool has the shape of the cavity to be cut. Cutting is derived from the cutting edges of abrasive grains flowing between the tool and workpiece. Figure 1 shows the important features of an ultrasonic machine.

A power source supplies current at ultrasonic frequencies to the acoustic head. Various vacuum - tube systems are in common use for this purpose.

The acoustic head contains the electromechanical converter, which drives the tool via a special holder (wave guide). The feed

mechanism applies the necessary force (5 - 8 kg) between the tool and workpiece. The abrasive feed system brings in fresh abrasive to the cutting area, carries the particles of material removed and cools the tool and workpiece.

In ultrasonic machining the material is removed by blows from grains of a harder abrasive. The abrasive particles are under control of a tool which vibrates with a small amplitude. Ofcourse, the abrasive also causes wear of the tool, but this is minimised by making the tool of a ductile material. The particles of abrasive are themselves split in the process and need to be gradually replaced by fresh abrasive carried in a liquid. The continuous flow of the slurry also serves to flush away the particles removed in the process. The material is removed in the form of small particles. As the tool vibrates at a high frequency and a large number of abrasive particles are active at a time, the rate of material removal is sufficient for practical purposes. Now because the material removal is based on the mechanism of brittle fracture of the workpiece, ultrasonic machining is particularly suitable for working on brittle materials.

The ultrasonic machining process has not yet been analysed to a sufficient degree of accuracy. The present investigation is intended to make a step forward for achieving accuracy in the analysis of the material removal rate in ultrasonic machining.

## CHAPTER II

## LITERATURE SURVEY

Miller [1] gave a semi-quantitative discussion on the rate of cutting in ultrasonic machining. He assumed that the particles are embedded in the workpiece and tool by the applied force and this causes plastic deformation and work hardening. The hardened parts are removed by chipping. The rate of material removal was calculated from the product of the total volume of plastically deformed material by the work of hardening per unit volume and the volume removed by chipping.

This theory is not accurate because of some of the assumptions that have been made. For instance, the dependence of the rate on work hardening implies that the treatment deals with plastic materials. But most materials that are worked by ultrasonic machining are brittle. The assumption of all the grains being cubes of the same size led him to deduce (incorrectly) that all grains take part in the cutting.

Shaw [2] assumed that material is removed primarily by two mechanisms due to

1. the direct impact of the tool on grains in contact with the workpiece and

2. the impact of grains accelerated by the tool. In both cases, the rate of material removal v is proportional to the volume V of material dislodged per impact, to the number of particles N making impacts per cycle and to the frequency f :

$$v \propto VNf \qquad (1)$$

Shaw assumed that the particles are identical spheres of diameter d equal to the mean grain size. Then the diameter D of the indentation is given by

$$D = 2 \left[dh\right]^{\frac{1}{2}} \qquad (2)$$

in which h is the depth of indentation. Then

$$v \propto D^3 \; Nf$$

$$\text{or} \qquad v \propto \left[dh\right]^{3/2} \; Nf \qquad (3)$$

The depth of indentation has been found by equating the mean static force to the mean force of impact of the tool on the grains. The force of impact of the tool has been assumed to have a linear relationship with time. The expression thus obtained is

$$h = \left[\frac{8 \, F_s \, y_0}{\pi \, NdH \, (1 + q)}\right]^{1/2} \qquad (4)$$

where

$F_s$ is the static force,

$y_0$ is the amplitude of vibration of the tool,

H is the hardness of the workpiece

and q is the ratio of the hardness of the workpiece to that of the tool.

The number of particles in the working gap is inversely proportional to the square of the diameter of each of the particle for a tool of fixed area; hence

$$N = \frac{k\,C}{d^2} \qquad (5)$$

in which k is the constant of proportionality and C is the concentration of the abrasive slurry.

Thus

$$h = \left[ \frac{8\,F\,y_0\,d}{\pi\,k\,HC\,(1+q)} \right]^{1/2} \qquad (6)$$

Shaw furthermore derived from calculations that only about 3% of the material removal is due to the impact of moving grains and the bulk of the material is removed by direct impact of the tool.

This theory has been based on a correct conception of the process as has been confirmed by subsequent experiments by high-speed cinematography [3]. All the same, Shaw did not obtain the correct result for the effects of frequency, amplitude and force [2]. In particular, the particles are irregular and only a few particles which stand out above others, take part in the material removal. The crushing of grains at high loads causes a fall in the rate of material removal. This cause and effect is not reflected in the theory.

Dikushin and Barke [4, 5] tried to relate the energy consumed in removing the material from the workpiece to the amplitude and force of vibration from the laws of conservation of energy and momentum. They considered the vibration of the mass of the concentrator, from the end of tool to the first displacement node of the concentrator, in response to an external force. The motion is sinosoidal upto the time of the impact. Moreover they assumed that the points of contact and separation are symmetricaly disposed with respect to the maximum displacement. The theory however failed to predict quantitatively the material removal rate.

Kazantsev [6] took into account the non-uniformity in the grain size. He assumed there are only some active grains with which the tool will contact. Taking a linear relationship between the portion of the active grits and the ratio ( $\delta/2R$) (where $\delta$ is the grain depth of indentation and R, the radius of the grains taken as spheres), he derived an expression for the material removal rate. But his theoretical results did not compare well with the experimental work done by others.

Rozenbarg [7] gave a qualitative analysis of the nature of damage in ultrasonic cutting. He assumed that the volume of damaged material is dependent on the maximal stress and the grain size. This means that the inhomogeneity in the grain size has a marked effect on the damage. He further gave a statistical distribution for an abrasive of any grain size based on experimental evidence:

$$F(d) = 1.095 \frac{N}{\bar{d}} \left[ 1 - \left( \frac{d}{\bar{d}} - 1 \right)^2 \right]^3 .$$

where

$\bar{d}$ is the mean diameter of the particles in the working gap.

$N$ is the number of particles in the working gap.

# CHAPTER III

## THEORETICAL ANALYSIS OF RATE OF MATERIAL REMOVAL IN ULTRASONIC MACHINING

The present theoretical analysis of the material removal rate in ultrasonic machining is made considering the direct impact of the abrasive grains on the workpiece. The inhomogeneity in the size of the grains is taken into consideration by the statistical distribution of grain size given by Rozenberg [7]. The grains are assumed to be spherical and the grain diameter is considered as the parameter denoting their size.

The material removal rate is determined from the amount of material removed by indentation of a single abrasive particle. The total material removal rate (v) has been arrived at by considering the active grains taking part in the process using the statistical distribution [7]. The expression involves the number of particles (N) in the working gap between the tool and workpiece, and the proximity (x) of the tool from the workpiece. Both these quantities depend on the diameter of the abrasive particles. For computational simplicity, the average number of particles in the working gap during the process and the mean distance of the tool from the workpiece have been considered.

## 3.1 Nature of Damage to the Material

The mechanism of material removal in ultrasonic machining is shown in Figure 2. The tool moves down from its mean position and at some point touches the largest grains, which are forced into the tool and workpiece. As the tool continues to move downwards, the force exerted on the grains increases as a result of which the grains may fracture. Eventually the tool comes to rest at a distance from the surface of the job corresponding to a certain maximum contact force as shown in Figure 2.

The distribution F (d) for the diameter d of the abrasive particles in the working gap is given by [7].

$$F(d) = 1.095 \frac{N}{\bar{d}} \left[ 1 - \left( \frac{d}{\bar{d}} - 1 \right)^2 \right]^3 \qquad (3.1)$$

where

N is the number of particles in the working gap between the tool and workpiece.

and $\bar{d}$ is the mean diameter of the particles in the working gap.

It has been assumed that the abrasive particles are incompressible. Hence the total depth of penetration by an abrasive particle of diameter d is

$$\delta_t + \delta_w = (d - x) \qquad (3.2)$$

where

x is the distance between the workpiece and the tool,

$\delta_t$ is the depth of indentation in the tool and

$\delta_w$ is the depth of indentation in the work piece.

Without the abrasive grains in the working gap, the tool motion is sinusoidal with amplitude $a_0$ as shown in Figure 2.

$$y = a_0 \sin \theta \tag{3.3}$$

Even though the force between the tool and the abrasive particles is effective only during a small portion of the cycle and may alter the free sinusoidal motion, however it is assumed that the motion of the tool remains sinusoidal with amplitude $a_0$ under the loaded conditions also.

The contact force acts only over the portion of the cycle between $\theta_1$ and $\pi/2$ as shown in Figure 2 causing plastic deformation in the tool and workpiece. Thus during this active portion of the cycle, the total depth of indentations is given by

$$\delta_w + \delta_t = y - y_s \tag{3.4}$$

where

$y_s$ is the distance the tool has moved downwards from its mean position before it touches the largest abrasive particle.

## 3.2 The Machining Rate

Figure 3 shows the geometry at the grit - work interface after indentation. By simple geometry, we obtain the relationship

$$r_w = \left[\delta_w ( d - \delta_w)\right]^{1/2}$$

$$= \left[\delta_w d \left(1 - \frac{\delta_w}{d}\right)\right]^{1/2}$$

For small values of $(\delta_w/d)$, the radius r of the contact indentation zone in the workpiece is

$$r_w = \left[\delta_w d\right]^{1/2} \qquad (3.5)$$

The volume $v_0$ fractured per grit is given by the whole of the hemi-spherical portion (shown shaded in Figure 3) during indentation 8 . Hence

$$v_0 = \frac{2}{3} \pi (\delta_w d)^{3/2} \qquad (3.6)$$

To determine $\delta_w$, we find out a relation between $\delta_w$ and $\delta_t$. Let $H_t$ and $H_w$ denote the brittle fracture hardness of the tool and workpiece respectively. The brittle fracture hardness is defined by the average contact stress for fracture (Figure 3). Hence

$$H_w = \frac{F_c}{\frac{\pi d}{2}\left[d - (d^2 - (2 r_w)^2)^{1/2}\right]}$$

Where $r_w$ is the radius of the contact indentation zone in the work-piece.

Now substituting the value of $r_w$ from equation (3.5) we get

$$H_w = \frac{F_c}{\frac{\pi d}{2}\left[d - (d^2 - 4 \;_w d)^{1/2}\right]}$$

$$= \frac{F_c}{\frac{\pi d}{2}\left[d - d\left(1 - \frac{4\delta_w}{d}\right)^{1/2}\right]}$$

$$= \frac{F_c}{\frac{\pi d}{2}\left[d - d + \frac{2\delta_w}{d} - \ldots\right]}$$

As $\frac{\delta_w}{d}$ is very small as compared to 1, hence its higher powers can be neglected. Thus

$$H_w = \frac{F_c}{\pi \delta_w d} \tag{3.7}$$

$$\left.\begin{aligned} \therefore \quad F_c &= k_w \delta_w \\ \text{where} \quad k_w &= \pi d H_w \end{aligned}\right\} \tag{3.8}$$

Similarly it can be derived that

$$\left.\begin{aligned} F_c &= k_t \delta_t \\ \text{where} \quad & \\ k_t &= \pi d H_t \end{aligned}\right\} \tag{3.9}$$

Dividing equation (3.8) by equation (3.9) we get

$$\frac{\delta_w}{\delta_t} = \frac{k_t}{k_w} = \frac{H_t}{H_w}$$

$$\therefore \quad \delta_w = q\,\delta_t \tag{3.10}$$

where

q is the hardness ratio, $\dfrac{H_t}{H_w}$

From equation (3.2)

$$\delta_t + \delta_w = (d - x)$$

or $$q\delta_w + \delta_w = (d - x)$$

$$\therefore \delta_w = \frac{(d - x)}{(1 + q)} \tag{3.11}$$

Now the material removal rate $V_d$ mm/min by abrasive grains of diameter d is written in terms of the volume removed per grit the number of abrasive particles of that diameter in the working gap and the frequency f as

$$V_d = v_0 F(d) f$$

The active grains taking part in the material removal process are those having diameters between x (the distance between the tool and workpiece) and $d_m$ (the maximum diameter of the abrasive grains). Hence the total material removal rate v mm/min considering all the effective particles of all the sizes is

$$V = \int_x^{d_m} v_0 \; F(d) f \, dd$$

$$= \int_x^{d_m} \frac{2}{3}\pi \, (\delta_w d)^{3/2} \; \frac{1.095 \, N}{\bar{d}} \left[1 - \left(\frac{d}{\bar{d}} - 1\right)^2\right]^3 f \, dd$$

Substituting the value of $\delta_w$ from equation (3.11) in the above expression we get

$$V = \frac{2\pi}{3} \frac{1.095\,N}{\bar{d}} \int_{x}^{d_m} \left\{ \frac{(d - x)\, d}{(1 + q)} \right\}^{3/2}$$

$$\left[ 1 - \left( \frac{d}{\bar{d}} - 1 \right)^2 \right]^3 f \, dd$$

or

$$V = \frac{2.29\,N}{(1 + q)\,\bar{d}} \int_{x}^{d_m} \left\{ \frac{(d - x)\, d}{1} \right\}^{3/2} \left[ 1 - \left( \frac{d}{\bar{d}} - 1 \right)^2 \right]^3 f \, dd \qquad (3.12)$$

The solution of the integral in the above expression is given in the Appendix.

The final result is as follows :

$$\bar{d}^{\,4} \left( 8\, I_7 - 12\, I_8 + 6\, I_9 - I_{10} \right) \Big|_{x}^{d_m} \qquad (3.13)$$

where

$$I_n = 2\, k^n \left[ \frac{(\sec b)^{2n-3} \tan^3 b}{(2n - 3)} - \frac{3\, (\sec b)^{2n-1} \tan b}{(2n - 3)\,(2n - 1)} \right.$$

$$+ \frac{3}{(2n-3)\,(2n-1)} \sum_{l=1}^{n} \frac{(2n)!\, \left[ (n - 1)! \right]^2}{2^l \left[ n! \right]^2 (2n-2l+1)!} (\sec b)^{2n-2l+1} \tan b$$

$$\left. + \frac{3}{(2n-3)\,(2n-1)} \frac{(2n)!}{2^n \left[ n! \right]^2} \ln\,(\sec b + \tan b) \right] \qquad (3.14)$$

$$n = 7,\ 8,\ 9,\ 10$$

$$k = x / \bar{d} \qquad (3.15)$$

$$b = \sec^{-1} (d/x)^{1/2}$$

### 3.3 The Average Number of Particles in the Working Gap

The average weight of an abrasive particle is found from the distribution function of the diameter of the abrasive particles. If the concentration of the abrasive slurry is given in terms of percentage by weight, the average number of abrasive particles in the working gap between the tool and the workpiece can be determined as follows :

If the diameter of the tool is D and the distance between the workpiece and the tool is $x_t$ at any instant of time, then the volume in the working gap containing the abrasive slurry is

$$V_G = \frac{\pi D^2}{4} x_t$$

Let C be the concentration of the abrasive slurry by weight (there are C kg. of abrasive in one kilogram of the liquid). If $P_a$ is the density of the abrasive and $P_f$ is the density of the liquid in the abrasive slurry, then in $\left( \frac{1}{P_f} + \frac{C}{P_a} \right) mm^3$ of abrasive slurry there is $\left( \frac{C}{P_a} \right) mm^3$ of abrasive.

Hence

$$V_a = \frac{C/P_a}{\left(\frac{1}{P_f} + \frac{C}{P_a}\right)} \qquad \frac{\pi D^2}{4} x_t$$

The weight of the abrasive particles in the working gap at any instant of time t will be :

$$W_t = V_a \times P_a = \frac{C}{\left(\frac{1}{P_f} + \frac{C}{P_a}\right)} \frac{\pi D^2}{4} x_t \qquad (3.16)$$

The number of particles in the working gap at any instant of time is

$$N_t = \frac{W_t}{w} \qquad (3.17)$$

where

w is the average weight of an abrasive particle.

Now the volume of an abrasive particle of diameter d is

$$v'(d) = \frac{4}{3} \pi (d/2)^3 = \frac{\pi}{6} d^3$$

The weight of this particle will be

$$W_d = \frac{\pi}{6} d^3 P_a$$

Hence the average weight of a particle will be

$$w = \int_{d_0}^{d_m} \frac{\pi}{6} d^3 P_a \frac{F(d)}{N} dd$$

$$= \frac{\pi}{6} P_a \frac{1.095}{\bar{d}} \int_{d_0}^{d_m} d^3 \left[1 - \left(\frac{d}{\bar{d}} - 1\right)^2\right]^3 dd$$

where

$d_0$ is the minimum diameter of the abrasive particle

$$\therefore \quad w = \frac{.057 P_a}{\bar{d}} \int_{d_0}^{d_m} d^3 \left[1 - \left(\frac{d}{\bar{d}} - 1\right)^2\right]^3 dd \qquad (3.18)$$

where

$$\int_{d_0}^{d_m} d^3 \left[ 1 - (\frac{d}{\bar{d}} - 1)^2 \right]^3 d\,(d) = -(\frac{1}{\bar{d}^6})(\frac{d_m^{10} - d_0^{10}}{10})$$

$$+ (\frac{d}{\bar{d}^5})(\frac{d_m^9 - d_0^9}{9}) - (\frac{12}{\bar{d}^4})(\frac{d_m^8 - d_0^8}{8})$$

$$+ \frac{8}{\bar{d}^3} (\frac{d_m^7 - d_0^7}{7}) \tag{3.19}$$

Thus substituting the value of $W_t$ from equation (3.16) and w from equation (3.18) into equation (3.17)

$$N_t = (\frac{C}{\frac{1}{P_f} + \frac{C}{P_a}}) (\frac{\pi D^2}{4}) (\frac{\bar{d}}{-057\ P_a})$$

$$(\frac{x_t}{\int_{d_0}^{d_m} d^3 \left\{ 1 - (\frac{d}{\bar{d}} - 1)^2 \right]^3 dd}) \tag{3.20}$$

Now from figure 2

$$x_t = x + (a_0 - y)$$

or $\quad x_t = x + a_0 (1 - \sin\theta)$

$$\therefore \quad x_t = (x + a_0) - a_0 \sin\theta \tag{3.21}$$

In equation (3.20), $N_t$ is a function of time only due to $x_t$. This working gap distance $(x_t)$ at any instant of time will decrease

with time. The average working gap distance in the period between the instant when the tool touches the largest abrasive grain and the time when it stops at a distance x from the workpiece is

$$\bar{X} = \int_{\theta_1}^{\pi/2} \left[ (x + a_0) - a_0 \sin\theta \right] d\theta \Big/ (\pi/2 - \theta_1)$$

where

$$\theta_1 = \sin^{-1}\left(\frac{y_s}{a_0}\right)$$

and $\quad y_s = (a_0 - d_m + x)$

$$\bar{X} = (x + a_0) - \frac{a_0 \cos\theta_1}{\left(\frac{\pi}{2} - \theta_1\right)} \tag{3.22}$$

Hence the average number of particles in the working gap can be given as

$$N = \left(\frac{C}{\frac{1}{P_f} + \frac{C}{P_0}}\right)\left(\frac{\pi D^2}{4}\right)\left(\frac{\bar{d}}{.057\, P_a}\right)\left(\frac{\bar{X}}{\int_{d_0}^{d_m} d^3 \left[1 - \left(\frac{d}{\bar{d}} - 1\right)^2\right]^3 dd}\right) \tag{3.23}$$

in which $\bar{X}$ is given by equation (3.22) and the value of integral by equation (3.19).

## 3.4 The Proximity of the Tool from the Workpiece

The distance X of the tool from the workpiece can be determined by equating the impulse due to the static force $F_s$ over the entire cycle to the impulse due to the variable contact force $F_c$ acting over the contact portion of the cycle [9].

The impulse due to the static force during the entire cycle will be

$$M_s = \frac{2\pi}{\omega} F_s \tag{3.24}$$

Again the impulse due to the contact force over the contact portion of the cycle will be

$$M_c = \int_{\theta_1/\omega}^{\pi/2\omega} k_w \delta_w \, dt$$

Substituting the value of $\delta_w$ from equation (3.11) we get

$$M_c = \int_{\theta_1/\omega}^{\pi/2\omega} \pi \, d \, H_w \, \frac{(d - x)}{(1 + q)} \, dt$$

From the equation (3.21) we know that the distance between the tool and workpiece at any instant of time will be

$$\frac{x}{t} = (x + a_0) - a_0 \sin \theta$$

This $x_t$ will also represent the diameter of the particle of abrasive contact with the tool at that instant of time.

Hence we get

$$M_c = \pi H_w \int_{\theta_1/\omega}^{\pi/2\omega} \left( \frac{d^2 - dx}{1 + q} \right) dt \tag{3.25}$$

$$\int_{\theta_1/\omega}^{\pi/2\omega} (d^2 - dx)\, dt = \frac{1}{\omega} \int_{\theta_1}^{\pi/2} (a_0^2 \sin^2 \theta - a_0 (x + 2a_0) \sin \theta$$

$$+ a_0 ( x + a_0))\, d\, \theta$$

$$= \frac{1}{\omega} \left( - \frac{a_0^2 \sin 2\, \theta}{4} + a_0 (x + 2a_0) \cos \theta \right.$$

$$\left. + a_0 (x + \frac{3}{2} a_0)\, \theta \right) \Big|_{\theta_1}^{\pi/2}$$

$$= \frac{1}{\omega} \left( \frac{a_0^2 \sin 2\, \theta_1}{4} - a_0 (x + 2a_0) \cos \theta_1 \right.$$

$$\left. + a_0 (x + \frac{3}{2} a_0) \quad ( \theta_1 - \pi/2 ) \right) \qquad (3.26)$$

Now equating (3.25) and (3.26)

$$M_s = M_c$$

We get

$$\frac{2\pi}{\omega} F_s = \frac{\pi H_w}{\omega(1+q)} \left( \frac{a_0^2 \sin 2\, \theta_1}{4} \right) - a_0 (x + 2a_0) \cos \theta_1$$

$$+ a_0 (x + \frac{3}{2} a_0) ( \theta_1 - \pi/2 ))$$

Or

$$\frac{2\, F_s\, ( 1 + q)}{H_w} = \left( \frac{a_0^2 \sin 2\, \theta_1}{4} - 2a_0 \cos \theta_1 + \frac{3}{2}\, a_0^2 \, ( \theta_1 - \pi/2) \right)$$

$$- ( a_0 \cos \theta_1 - a_0 ( \theta_1 - \pi/2))\, x$$

$$\frac{2(1+q)\,F_s}{} = \left(a_0 \sin 2\theta_1 - 2a_0 \cos\theta_1 + \frac{3}{2}\,a_0^2\,(\theta_1 - \pi/2)\right)$$

$$- a_0\,(\cos\theta_1 - (\theta_1 - \pi/2))\,x \qquad (3.27)$$

where

$$\theta_1 = \sin^{-1}\left(\frac{a_0 + x - d_m}{a_0}\right)$$

CHAPTER IV

EXPERIMENTAL SET-UP AND THE RESULTS OF EXPERIMENTS

The experiments in the present investigation have been conducted on 200 watt Cavitron Ultrasonic Drilling Machine to study the effect of frequency, concentration of abrasive slurry and static force on the material removal rate.

## 4.1 Description of the Experimental Set-up

The different instruments in the experimental set-up are shown in the photograph in Figure 4.

The power supply to the machine is from a Vacuum-tube generator. Input power to the generator is from a single phase, 60 cps 120 volts power supply. The generator consists of an electronic oscillator which sets up an alternating current at low power and an amplifier which increases the power to produce mechanical oscillations in the transducer.

The transducer is a unit in the ultrasonic machine where conversion of electrical energy to mechanical energy takes place. The transducer consists of a special shaped magnetostrictive material surrounded by a coil wire carrying a low voltage D.C. polarizing current and a high frequency alternating magnetic field. The conversion of energy takes place within this material, producing minute linear changes in length in step with the alternating field.

FIGURE 4. EXPERIMENTAL SET-UP

The magnetostrictive material is brazed to a connecting body called the wave guide. The wave guide is designed to receive, transmit and amplify the length changes in the transducer material. A removable tool holder and tool, continues the transmission, further amplifying the stroke at the tool tip.

In effect, the entire length of these elements react in much the same manner as a rubber band being alternately stretched and allowed to return to its normal length at a very rapid frequency.

A recirculating pumping unit supplies abrasive slurry to the tool and workpiece. The transducer is cooled by a water coolant system in the head of the machine.

## 4.2 Experimental Procedure and Results

The amplitude of the tool motion is measured by a calibrated microscope attached to the machine. The frequency of the tool motion is determined by a frequency counter. The penetration of tool in the workpiece is measured from a dial on the machine.

Boron Carbide of 400 mesh has been used as abrasive with water, as the slurry for machining.

Three experiments were conducted on the above set-up:

A. To study the effect of static force on rate of machining, the frequency (25.5 Kc), amplitude (.0625 mm) and concentration of the abrasive slurry (.25) were kept constant. The static

load was increased in steps, and at each stage the rate of penetration of the tool in the work-piece (mm/min) noted from the dial reading in the machine. The result has been given in the graph in Figure 5.

B. The effect of frequency on the rate of machining was studied by keeping the amplitude (.0625mm), the concentration of the abrasive slurry (.25) and the static load (.454 kg) constant. The frequency was varied by the tuning device in the oscillator and the rate of penetration of the tool in the work-piece for each value of frequency. Figure 6 gives the result in the graph.

C. The frequency (25.5 Kc), amplitude (.0625 mm) and static load (.454 Kg) were kept constant to study the effect of concentration on rate of machining. The concentration of the abrasive slurry was varied from .125 to .5 in steps of .125. At each step the rate of penetration of the tool in the workpiece taken down. The result has been plotted in the graph in Figure 7.

In figure 5,6 and 7 the theoretical curves have also been plotted from the equations (3.13), (3.14), (3.15), (3.23) and (3.27) using the following values of the constants:

1. Hardness of tool = 150 $Kg/mm^2$
2. Hardness of workpiece = 600 $Kg/mm^2$
3. Density of abrasive (Boron carbide)= $4.0 \times 10^{-6}$ $Kg/mm^3$
4. Maximum Diameter of abrasive (400 mesh) = .047 mm
5. Minimum Diameter of abrasive (400 mesh ) = .032 mm

The first constant has been taken from the manual with the machine, the second and third from "Engineering Materials' Handbook" and the last two constants from "Soil Mechanics" by James E. Bowles.

CHAPTER V

DISCUSSIONS

There is an optimum material removal rate corresponding to a certain static load as can be seen from the experimental curve in Figure 5. The theoretical curve in the figure depicts an indefinite increase in the material removal rate as the static force increases. The reasons for this discrepancy are as follows:

1. The crushing abrasive grains as the tool moves downwards has not been considered in the analysis. This phenomenon is particularly responsible for the fall in the material removal rate at higher static loads.

2. The analysis assumes, the amplitude of the sinusoidal tool motion under loaded conditions same as that in the case of free sinusoidal motion of the tool. In actual conditions the amplitude of the tool motion may change when the abrasive particles are introduced in the working gap. The cutting rate decreases as the amplitude decreases [7].

From the experimental curve in Figure 6, it is seen that the relationship between the cutting rate and the frequency is linear. The theoretical result also shows similar trend but the values are higher than the experimental ones. This is because the analysis does not consider the effect of change in statistical distribution of the abrasive particles in the working gap due to the crushing of grains as the tool moves downwards.

The experimental rate of material removal increases with the increase in concentration of the abrasive slurry (Figure 7). The rate of cutting however becomes asymptotic near about a concentration of .50 . The experiments were conducted in the range of the resonant frequency (24.8 Kc. to 25.5 Kc.) because outside this range the cutting rate is very slow and difficult to measure. The theoretical curve shows a higher rate of increase in the cutting rate with increase in the concentration of the abrasive slurry and does not depict the asymptotic behaviour. The reasons for this are :

(1) In the analysis, the effect of increasing the packing density of the abrasive particles in the working gap has not been considered. For example, if the concentration is 100% by weight, there will be either no cutting at all or else it will be very negligible.

(2) The whole analysis in the present dissertation has been based on only the direct impact on the abrasive grains by the tool. The material removal due to the impact of the abrasive grains accelerated by the tool has not been taken into account.

# CHAPTER VI

## CONCLUSIONS AND SUGGESTIONS

### 6.1 Conclusions

The optimum rate of material removal using 400 mesh Boron Carbide, at a frequency of 25.5 Kc and amplitude of .0625 mm, for glass is 0.6502 mm/min corresponding to a static force of 0.32 kg.

The theoretical curve between the rate of penetration of the tool into the workpiece and static force shows the same trend as the experimental curve for static force below 0.40 kg. After this point there is a gradual increase in the theoretical rate as the static force increases where as the experimental curve shows a fall in the rate of material removal with increase in static force.

Both the theoretical and experimental results confirm a linear relationship between the rate of material removal and the frequency of vibration of the tool.

The rate of material removal increases with the increase in concentration of the abrasive slurry, but near about a concentration of .50 : the experimental curve becomes asymptotic. The theoretical results depict the increasing trend but do not show the asymptotic behaviour.

### 6.2 Suggestions

The optimum rate of material removal for various materials can be found with different abrasives. The results will give the

best cutting conditions for the various materials.

The material removal due to the impact of moving grains accelerated by the tool can be incorporated in the analysis to give better results.

During the cutting process, abrasive slurry can be taken out after a fixed interval of time, dried up and the size distribution of the abrasive particles determined. This abrasive can be used again for cutting. Thus in this way, the change in the statistical distribution of the abrasive particles can be determined as a function of time. The cutting rate can then be analysed better. Moreover, the time for which the abrasive slurry can be used continuously for cutting effectively is also found out.

The amplitude change in the tool motion, when the abrasive particles are introduced in the working gap, can also be determined theoretically as well as experimentally to do a better analysis of the material removal rate.

# APPENDIX

## Solution of Integral in Equation 3.12

$$\int_x^{d_m} \left[d^2 - dx\right]^{3/2} \left[1 - (d/\bar{d} - 1)^2\right]^3 d\,(d) \qquad (1)$$

$$= \bar{d}^3 \int_x^{d_m} \left[(d/\bar{d})^2 - \frac{d}{\bar{d}}\ \frac{x}{\bar{d}}\right]^{3/2} \left[1 - (d/\bar{d} - 1)^2\right]^3 d\,(d)$$

Let

$$u = d/\bar{d} \quad \text{and} \quad k = x/\bar{d}$$

$$\bar{d}^4 \int_x^{d_m} \left[u^2 - ku\right]^{3/2} \left[1 - (u - 1)^2\right]^3 du$$

$$= \bar{d}^4 \int_x^{d_m} \left[u^2 - ku\right]^{3/2} \left[8u^3 - 12u^4 + 6u^5 - u^6\right] du$$

$$= 8\,\bar{d}^4 \int_x^{d_m} u^3 \left[u^2 - ku\right]^{3/2} du - 12\,\bar{d}^4 \int_x^{d_m} u^4 \left[u^2 - ku\right]^{3/2}$$

$$du + 6\,\bar{d}^4 \int_x^{d_m} u^5 \left[u^2 - ku\right]^{3/2} du - \bar{d}^4 \int_x^{d_m} u^6 \left[u^2 - ku\right]^{3/2} du$$

Let us evaluate the general integral

$$\int u^{\bar{n}} \left[u^2 - ku\right]^{3/2} du$$

where

$$\bar{n} = 3,\ 4,\ 5,\ 6$$

We make the following substitution

$$u = k \sec^2 b$$

$$du = 2k \sec^2 b \tan b\, db$$

Then, the integral becomes

$$\int k^{\bar{n}} \sec^{2\bar{n}} b \left[ k^2 \sec^2 b \tan^2 b \right]^{3/2} 2k \sec^2 b \tan b \, db$$

$$= 2k^{\bar{n}+4} \int (\sec b)^{2\bar{n}+5} (\tan b)^4 \, db$$

$$= P \int (\sec b)^m (\tan b)^4 \, db$$

where

$$m = 2\bar{n} + 5$$

$$m = 11, 13, 15, 17$$

$$P = 2k^{\bar{n}+4}$$

$$P \left[ \int \tan^3 b \left\{ (\sec b)^{m-1} \sec b \tan b \right\} db \right]$$

$$= P \left[ \frac{\tan^3 b \sec^m b}{m} - \frac{1}{m} \int \sec^m b \; 3 \tan^2 b \sec^2 b \, db \right]$$

$$= P \left[ \frac{\tan^3 b \sec^m b}{m} - \frac{3}{m} \int \tan b \left\{ \sec^{m+1} b \sec b \tan b \right\} db \right]$$

$$= P \left[ \frac{\tan^3 b \sec^m b}{m} - 3 \frac{\tan b \sec^{m+2} b}{m(m+2)} \right.$$

$$\left. + \frac{3}{m(m+2)} \int \sec^{m+4} b \, db \right]$$

We solve the integral in the above expression as follows :

$$\int \sec^{m+4} b \, db = \int (\sec b)^{m+2} (1 + \tan^2 b) \, db$$

$$= \int (\sec b)^{m+2} \tan^2 b \, db + \int (\sec b)^{m+2} \, db$$

$$= \int \tan b \left[ (\sec b)^{m+1} \sec b \tan b \right] db + \int (\sec b)^{m+2} \, db$$

$$= \frac{\tan b \, (\sec b)^{m+2}}{(m+2)} - \frac{1}{(m+2)} \, (\sec b)^{m+2} \sec^2 b \, db + (\sec b)^{m+2} \, db$$

$$\frac{m+3}{m+2} \int \sec^{m+4} b \, db = \frac{(\sec b)^{m+2} \tan b}{(m+2)} + \int (\sec b)^{m+2} \, db$$

$$\therefore \int (\sec b)^{m+4} \, db = \frac{(\sec b)^{m+2} \tan b}{(m+3)} + \frac{m+2}{m+3} \int (\sec b)^{m+2} \, db$$

$$\int (\sec b)^{m+2} \, db = \frac{(\sec b)^{m} \tan b}{(m+1)} + \frac{m}{m+1} \int (\sec b)^{m} \, db$$

$$\int (\sec b)^{m} \, db = \frac{(\sec b)^{m-2} \tan b}{(m-1)} + \frac{m-2}{m-1} \int (\sec b)^{m-2} \, db$$

$$\int (\sec b)^{m-2} \, db = \frac{(\sec b)^{m-4} \tan b}{(m-3)} + \frac{m-4}{m-3} \int (\sec b)^{m-4} \, db$$

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$$\int (\sec b)^{m-21+8} \, db = \frac{(\sec b)^{m-21+6} \tan b}{m-21+7} + \frac{m-21+6}{m-21+7} \int (\sec b)^{m-21+6} \, db$$

$$(\sec b)^{m-21+6} \, db = \frac{(\sec b)^{m-21+4} \tan b}{m-21+5} + \frac{m-21+4}{m-21+5} \int (\sec b)^{m-21+4} \, db$$

If $(m + 4)$ is odd (which is the case) then

$$\int (\sec b)^3\, db = \frac{\sec b \tan b}{2} + \frac{1}{2}\int \sec b\, db$$

$$\int \sec b \quad db = \ln (\sec b + \tan b)$$

Therefore, we have

$$\int (\sec b)^{m+4}\, db = \frac{(\sec b)^{m+2} \tan b}{(m+3)} + \frac{(m+2)}{(m+3)(m+1)} (\sec b)^m \tan b$$

$$+ \frac{(m+2)\, m}{(m+3)(m+1)(m-1)} (\sec b)^{m-2} \tan b$$

$$+ \frac{(m+2)\, m\, (m-2)}{(m+3)(m+1)(m-1)(m-3)} (\sec b)^{m-4} \tan b + ----$$

$$+ \frac{(m+2)\, m\, (m-2) --- (m-2l+6)}{(m+3)(m+1)(m-1)(m-3) -- (m-2l+5)}$$

$$(\sec b)^{m-2l+4} \tan b + -------$$

$$+ \frac{(m+2)\, m\, (m-2) --- (m-2l+6) --- 3}{(m+3)(m+1)(m-1) --- (m-2l+5) --- 2}$$

$$\left[\sec b \tan b + \ln (\sec b + \tan b)\right]$$

We can reduce the coefficient of $l^{th}$ term to the following forms

$$\frac{(m+2)\, m\, (m-2) ----- (m-2l+6)}{(m+3)(m+1)(m-1)(m-3) --- (m-2l+5)}$$

Since $(m+4)$ is odd, we can write $m = 2n-3$

where $n = 7, 8, 9, 10$

Substituting this value of m in the coefficient of $l^{th}$ term

$$\frac{(2n)\,!}{2^{l}\left[n\,(n-1)\,(n-2)\,(n-3)\,\ldots\,(n-l+1)\right]^{2}\,(2n-2l+1)\,!}$$

$$= \frac{(2n)\,!\,\left\{(n-l)\,!\right\}^{2}}{2^{l}\left\{n\,!\right\}^{2}\,(2n-2l+1)\,!}$$

$$(\sec b)^{2n+1}\,db = \sum_{l=1}^{n} \frac{(2n)\,!\,\left[(n-l)\,!\right]^{2}}{2^{l}\left\{n\,!\right\}^{2}\,(2n-2l+1)\,!}\,(\sec b)^{2n-2l+1}\tan b$$

$$+ \frac{(2n)\,!}{2^{l}\left\{n\,!\right\}^{2}}\ln\,(\sec b + \tan b)$$

Now, we have

$$m = 2\,\bar{n} + 5$$

$$m = 2n - 3$$

$$\therefore n = \bar{n} + 4$$

$$\int u^{\bar{n}}\left[u^{2} - ku\right]^{3/2}\,du = 2k^{n}\left[\frac{(\sec b)^{2n-3}\,\tan^{3} b}{(2n-3)} - \frac{3\,(\sec b)^{2n-1}\,\tan b}{(2n-3)\,(2n-1)}\right.$$

$$+ \frac{3}{(2n-3)\,(2n-1)}\sum_{l=1}^{n}\frac{(2n)\,!\,\left[(n-l)\,!\right]^{2}}{2^{l}\left[n\,!\right]^{2}\,(2n-2l+1)\,!}$$

$$(\sec b)^{2n-2l+1}\,\tan b$$

$$\left.+ \frac{3}{(2n-3)\,(2n-1)}\,\frac{(2n)\,!}{2^{n}\left[n\,!\right]^{2}}\,2\,\ln\,(\sec b + \tan b)\right\}$$

Hence the solution of the given integral is

$$\bar{d}^{\,4}\,(8\,I_{7} - 12\,I_{8} + 6\,I_{9} - I_{10})\,\Big|_{x}^{d_m}$$

Where

$$I_n = 2\,k^n \left[ \frac{(\sec b)^{2n-3} \tan^3 b}{(2n-3)} - \frac{3\,(\sec b)^{2n-1} \tan b}{(2n-3)\,(2n-1)} \right.$$

$$+ \frac{3}{(2n-3)\,(2n-1)} \sum_{l=1}^{n} \frac{(2n)!\,\left[(n-1)!\right]^2}{2^l \left[n!\right]^2 (2n-2l+1)!} (\sec b)^{2n-2l+1} \tan b$$

$$\left. + \frac{3}{(2n-3)\,(2n-1)} \frac{(2n)!}{2^n \left[n!\right]^2} \ln(\sec b + \tan b) \right]$$

$n = 7, 8, 9, 10$

$k = x / \bar{d}$

$b = \sec^{-1} \left[ (d / x)^{1/2} \right]$

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